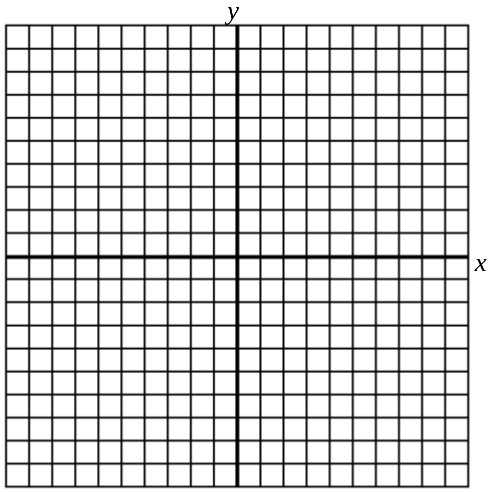
Investigation 53 – Graphing Rationals   Name:________________________

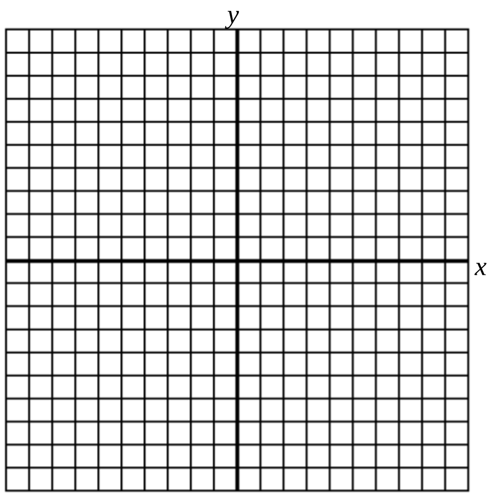
The graph of an inverse variation function, $f(x) = \dfrac{1}{x}$, looks like:



Vertical Asymptote:_________________

Horizontal Asymptote: _________________

We can transform it to: $g(x) = f(x-2) + 3 = \frac{1}{x-2} + 3$



Vertical Asymptote:_________________

Horizontal Asymptote: _________________

We can also re-write $g(x)$ as a single fraction by getting an LCD.

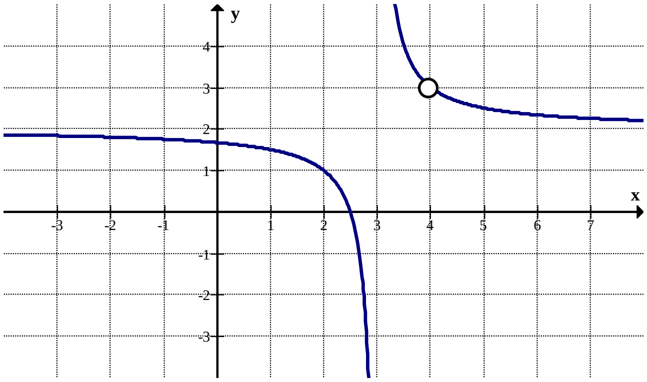
$g(x) = \frac{1}{x-2} + 3 =$

When we write the equation this way we consider it a "Rational Function."

A R_______________ F______________ is defined as a function that is a ratio of two polynomials $\dfrac{p(x)}{q(x)}$ such that the degree of $q(x)$ is at least 1.

We are going to look at functions presented this way to do a full analysis of their graphs.

Identify the characteristics of the graph of $h(x) = \frac{(2x-5)(x-4)}{(x-3)(x-4)}$.



equations(s) of vertical asymptote(s):__________

equations(s) of horizontal asymptote(s):________

coordinates of hole:_______________

domain:_________________________

range:__________________________

What are the "excluded values" of $h(x)$? (Remember, these are values that make the denominator ___.)

How does the domain relate to the excluded values?

What occurs at each of these excluded values?

Discontinuities can be places where the graph has vertical asymptotes or holes. How do the discontinuities relate to the excluded values?

Which discontinuities are removable? Why does this occur algebraically?

Which discontinuities are non-removable? Why does this occur algebraically?

Division by zero creates problems in the graphs of rational functions. When a denominator equals zero, two things can happen in the graph. The graph will have either a hole or a vertical asymptote at these values of *x*. Holes are plotted on a graph as open circles. Asymptotes are plotted as dashed lines.

Holes: If a factor can be reduced from the top and bottom of a rational function, the graph will have a hole. This is because at that one specific *x*-value, the function will have division by zero and will not exist. For any other *x*-values, however, the factor will be insignificant. The *x*-value of a hole is the *x*-value that creates 0/0. The *y*-value of the hole is the *y*-value that you get when you plug the *x*-value into the REDUCED function.

Holes are _______________ or _________________ discontinuities.

Vertical Asymptotes: Given a rational function in reduced form, a vertical asymptote will occur at any value of *x* that results in division by zero. This is due to the fact that as the denominator gets closer and closer to zero in value, the *y*-value of the function gets larger and larger. ($y \to \infty$ or $y \to -\infty$) The equation for a vertical asymptote is x = the number that caused division by zero. It is the zero of a factor that cannot be reduced.

Vertical Asymptotes are _______________ or _________________ discontinuities.

The Domain and Range are the possible values for x and y in a function.

Domain: Division by zero creates problems, so the domain of a rational function is $\{x \neq$ any zeros of the denominator$\}$ in the original problem (before reducing).

Range: For the range, you need to look at the graph and pay attention to any local maxima/minima, holes, and horizontal asymptotes. Remember, if a y-value exists anywhere on the graph, it is part of the range.

Zero is a powerful number. Zero can appear three ways in a rational function.

$\frac{number}{0}$ When the _______________of the fraction is zero, the fraction does not exist. Thus, you get ___________________ in the graph.

$\frac{0}{number}$ When the _______________ of the fraction is zero, the fraction simplifies to _____. Thus, you get ___________________ in the graph.

$plug\ in\ x = 0$ When you plug in $x = 0$ to the function, you can usually simplify to a number, y. The coordinates would be $(0, y)$. Thus, you get ___________________ in the graph.

The x and y-intercepts of a graph tell where the curve hits the axes. You can find intercepts for rational functions just as you would for any function.

***x*-intercepts** Every point on the x-axis has a y-value of zero. So, to find the x-intercepts of a curve, set $y = 0$ and solve for x. This means that the top of the fraction equals zero. Remember to use the reduced form of the equation, because a hole cannot be an intercept.

***y*-intercepts** Every point on the y-axis has an x-value of zero. So, to find the y-intercepts of a curve, plug in $x = 0$ and solve for y. Remember to use the reduced form of the equation, because a hole cannot be an intercept.

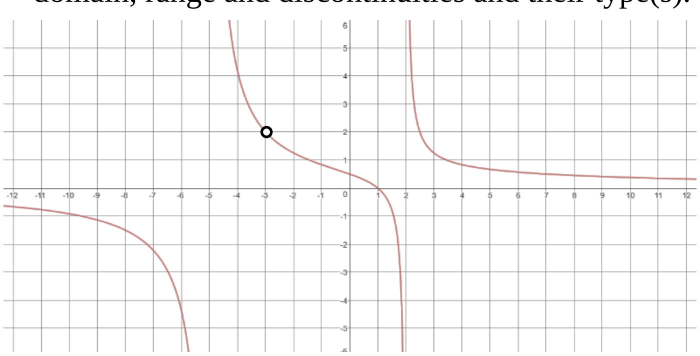
Example: Identify the characteristics of $k(x) = \frac{5(x-1)(x+3)}{(x-2)(x+3)(x+5)}$ using algebra.

equations(s) of vertical asymptote(s):__________

coordinates of hole:____________

x-intercept(s):__________ y-intercept(s): __________

Now confirm your answers with the graph of $k(x)$ given below. Also state the horizontal asymptote, domain, range and discontinuities and their type(s).



equations(s) of horizontal asymptote(s):________

domain:_________________________

range:___________________________

discontinuities:____________________