

# Investigation Geometric Sequences Answers

## Investigation Geometric Sequences

Name: \_\_\_\_\_

A **geometric sequence** is a sequence with a constant **common ratio**, or, in plain language, where you multiply by a certain number to get from term to term.

Example: 1, 2, 4, 8, 16, 32, 64, ...      common ratio =  $r = 2$

Example: 12,  $-4\frac{4}{3}$ ,  $-\frac{4}{9}$ ,  $\frac{4}{27}$ , ...      common ratio =  $r = -\frac{1}{3}$

For #1 & 2: a) Determine whether the sequence is geometric or not,  
b) find the common ratio, and  
c) list the next three terms

1)  $2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

a) Geometric? yes      b)  $r = \frac{1}{2}$       c)  $\frac{1}{16}$ ,  $\frac{1}{32}$ ,  $\frac{1}{64}$

2) -4, 12, -36, 108, ...

a) Geometric? yes      b)  $r = -3$       c) -324, 972, -2916

Missing terms in a geometric sequence are called **geometric means**.

Example: Find the geometric mean of 4 and 36:

4, 12, 36  
or  
-12

Since we know this is a geometric sequence, the pattern is to *multiply* by a constant each time.

$$4 \cdot r^2 = 36 \quad r = \pm 3$$

$$r^2 = 9$$

3) Find three geometric means of 2 and 162.

2    —    —    —    162

6, 18, 54

OR

-6, 18, -54

$2r^4 = 162$   
 $r^4 = 81$   
 $r = \pm 3$

4) Find two geometric means of -18 and  $\frac{9}{4}$ .

-18    —    —     $\frac{9}{4}$

$9, \frac{9}{2}$

$-18r^3 = \frac{9}{4}$   
 $r^3 = -\frac{1}{8}$   
 $r = -\frac{1}{2}$

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## Recursive and Explicit Formulae –

The **recursive formula** for a geometric sequence is:

$$a_n = a_{n-1} \cdot r \quad \text{where } a_{n-1} \text{ is the previous term}$$

$r$  is the common ratio

Example:  $\begin{cases} a_1 = 12 \\ a_n = \frac{1}{3} a_{n-1}, \text{ for } n \geq 2 \end{cases}$

What are the first 5 terms? 12, 4,  $\frac{4}{3}$ ,  $\frac{4}{9}$ ,  $\frac{4}{27}$

The **explicit formula** for a geometric sequence is:

$$a_n = a_1 \cdot r^{n-1} \quad \text{where } a_1 \text{ is the first term,}$$

$n$  is the number of the term that you want  
 $r$  is the common ratio

Example:  $a_n = (5)2^{n-1}$

What are the first 5 terms? 5, 10, 20, 40, 80

What is the 21<sup>st</sup> term? 5,242,880

$$a_{21} = 5(2)^{20}$$

Write a recursive and an explicit formula for each:

5)  $15, 3, \frac{3}{5}, \dots$

$r = \frac{1}{5}$

Recursive:  $\begin{cases} a_1 = 15 \\ a_n = \frac{1}{5} a_{n-1}, n \geq 2 \end{cases}$

Explicit:  $a_n = 15 \left(\frac{1}{5}\right)^{n-1}$

6)  $8, -4, 2, -1, \dots$

$r = -\frac{1}{2}$

Recursive:  $\begin{cases} a_1 = 8 \\ a_n = -\frac{1}{2} a_{n-1}, n \geq 2 \end{cases}$

Explicit:  $a_n = 8 \left(-\frac{1}{2}\right)^{n-1}$

7) If the fifth term of a geometric sequence is 6, and the ninth term is 96,

a) What is the common ratio?

$r = 2$  or  $r = -2$

$$\frac{6}{5} \text{ --- } \frac{96}{9}$$

b) What is the first term?

$a_1 = \frac{3}{8}, \frac{3}{4}, \frac{3}{2}, 3, 4, 5, 6$

$6r^4 = 96$   
 $r^4 = 16$   
 $r = \pm 2$

c) What is the recursive formula for the sequence?

$\begin{cases} a_1 = \frac{3}{8} \\ a_n = 2a_{n-1}, n \geq 2 \end{cases}$  OR  $\begin{cases} a_1 = \frac{3}{8} \\ a_n = -2a_{n-1}, n \geq 2 \end{cases}$

d) What is the explicit formula for the sequence?

$a_n = \frac{3}{8}(2)^{n-1}$  OR  $a_n = \frac{3}{8}(-2)^{n-1}$