**Investigation Arithmetic Sequences** **Name:** _____

A **sequence** is an ordered list that follows a set pattern.

An **arithmetic sequence** is a sequence with a constant **common difference**, or, in plain language, where you add a certain number to get from term to term.

Example: 2, 4, 6, 8, 10, 12, . . . common difference = $d = 2$

Example: 10, 4, -2, -8, -14, . . . common difference = $d = -6$

For #1 & 2: a) Determine whether the sequence is arithmetic or not,
b) find the common difference, and
c) list the next three terms

1) 11, 8, 5, 2, . . . -3

a) Arithmetic? yes b) $d =$ -3 c) -1, -4, -7

2) 5, 10, 15, 20, . . . +5

a) Arithmetic? yes b) $d =$ 5 c) 25, 30, 35

Missing terms in an arithmetic sequence are called **arithmetic means**.

Example: Find the arithmetic mean of 13 and 19:

13, 16, 19 Since we know this is an arithmetic sequence, the pattern is to *add* a constant each time.

13, ___, 19

$13 + 2d = 19$

$2d = 6$

$d = 3$

3) Find three arithmetic means of 2 and -22.

2, -4, -10, -16, -22

$2 + 4d = -22$

$4d = -24$ $d = -6$

4) Find four arithmetic means of -18 and 62.

-18, -2, 14, 30, 46, 62

$-18 + 5d = 62$

$5d = 80$

$d = 16$

**Recursive and Explicit Formulae and Notation** –

The $n^{th}$ term of a sequence is written $a_n$. $n$ is the number of the term and $a_n$ is the actual term.

In the sequence 2, 4, 6, 8, 10, 12

$a_1 =$ 2 $a_2 =$ 4

Fill in the missing term names in the sequence $a_1, a_2, a_3, \ldots,$ $a_{n-1}$, $a_n$, $a_{n+1}, \ldots$

A **recursive formula** for a sequence is a rule that defines a new term, based on previous terms.

Example: To get the $n^{th}$ term of the sequence 2, 5, 8, 11, . . . we can use the rule $a_n = a_{n-1} + 3$ because to get a term, you add 3 to the previous term.

So, to find the 4th term, we take the 3rd term and add 3. $a_4 = a_3 + 3 = 8 + 3 = 11$

The **recursive formula** for an arithmetic sequence is:

$a_n = a_{n-1} + d$ where $a_{n-1}$ is the previous term
$d$ is the common difference

Example: $a_1 = 12$
$a_n = a_{n-1} - 7$; for $n \geq 2$

What are the first 5 terms? 12, 5, −2, −9, −16

An **explicit formula** for a sequence is like a function where you plug in any number $n$ to get the $n^{th}$ term.

Example: To get the $n^{th}$ term of the sequence 2, 5, 8, 11, . . . we can use the rule $a_n = 2 + (n-1)(3)$, because the sequence starts at 2 and the difference between the terms is 3 (like a slope).

The **explicit formula** for an arithmetic sequence is:

$a_n = a_1 + (n-1)\,d$ where $a_1$ is the first term,
$n$ is the number of the term that you want
$d$ is the common difference.

Example: $a_n = 15 + (n-1)(-5)$

What are the first 5 terms? 15, 10, 5, 0, −5

What is the 21st term? −85

$a_{21} = 15 + (21-1)(-5) = 15 + (20)(-5) = -85$

Write a recursive and an explicit formula for each:

5) 3, 7, 11, 15, . . .

Recursive: $a_n = a_{n-1} + d$ Explicit: $a_n = a_1 + (n-1)(d)$

$\begin{cases} a_1 = 3 \\ a_n = a_{n-1} + 4 \end{cases}$

$a_n = 3 + (n-1)(4)$

6) 13, 7, 1, -5, . . .  $d=-6$

Recursive:

$$\begin{cases} a_1 = 13 \\ a_n = a_{n-1} - 6 \end{cases}$$

Explicit:

$$a_n = 13 + (n-1)(-6)$$

7) $a_n = 25 + (n-1)(10)$

a) What are the first 5 terms? 25, 35, 45, 55, 65

b) What is the 11th term? 125

$$a_{11} = 25 + (11-1)(10) = 125$$

8) $\begin{cases} a_1 = 12 \\ a_n = a_{n-1} - 6; \text{ for } n \geq 2 \end{cases}$

What are the first 5 terms? 12, 6, 0, -6, -12

$d=-6$

9) If the fifth term of an arithmetic sequence is 6, and the eleventh term is 36

a) What is the common difference?

$d=5$

b) What is the first term?

$-14$

c) What is the recursive formula for the sequence?

$$\begin{cases} a_1 = -14 \\ a_n = a_{n-1} + 5 \end{cases}$$

d) What is the explicit formula for the sequence?

$$a_n = -14 + (n-1)(5)$$

-14, -9, -4, 1, 6, 11, 16, 21, 26, 31, 36 (+d; 5 steps from 6 to 36; 11 terms)

$6 + 6d = 36$

$6d = 30$

$d = 5$

$a_n = a_1 + (n-1)d$

$6 = a_1 + (5-1)(5)$

$6 = a_1 + 20$

$-14 = a_1$