Investigation – Increasing/Decreasing & End Behavior Name:_______________________

Increasing/Decreasing/Constant

To describe where a function is increasing, decreasing or constant, we always work from ***left to right***.

- If the y's are getting bigger, the function is **increasing**.
- If the y's are getting smaller, the function is **decreasing**.
- If the y's stay the same, the function is **constant**.

End Behavior

Right

To describe a function's end behavior, we look at what the graph is doing on the far left and the far right.

- For the **right end behavior** we consider very big positive x's, or "as x approaches infinity." We use the notation "as $x \to \infty$". If the y's go up, we say y also approaches infinity, or in symbols $y \to \infty$. If the y's go down, we say y approaches negative infinity, or in symbols $y \to -\infty$.

  as $x \to \infty, y \to \infty$
- For the **left end behavior** we consider very big negative x's, or "as x approaches negative infinity." We use the notation "as $x \to -\infty$". If the y's go up, we say $y \to \infty$. If the y's go down, we say $y \to -\infty$.

  as $x \to \infty, y \to -\infty$

  as $x \to -\infty, y \to \infty$
- If a graph has an endpoint (no arrow), it does not have an end behavior in that direction.

<u>Steps for analyzing a graph:</u>

$x \to -\infty, y \to -\infty$

- Label any end points, turning points, and arrows with their coordinates. When labeling arrows, we often need to use ∞ and $-\infty$.
- To determine the x-intervals for increasing, decreasing and constant, break the graph into zones as you travel from left to right. Then, describe each zone by its **x-values.** (The y-coordinates are never used!) The intervals are always open (no equal to) and are written like this:

  left x-value $<x<$ *right x-value* or (*left x-value, right x-value*)
- The end behavior is "as $x \to$___, $y \to$___" where the blanks are filled in by the "coordinates" of the arrows.

<u>Example 1:</u> On the right is the graph of $f(x) = x^2 + 2$.

Describe the following:

Increasing: $(0, \infty)$ $0 < x < \infty$

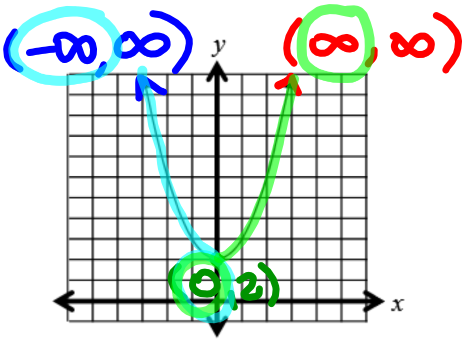
Decreasing: $(-\infty, 0)$ $-\infty < x < 0$

Constant: never

End Behavior:

As $x \to \infty$, $y \to$ ∞.

As $x \to -\infty$, $y \to$ ∞.



<u>Example 2:</u> On the right is the graph of $g(x)$. Describe the following:

Increasing: $(2, \infty)$ $2 < x < \infty$

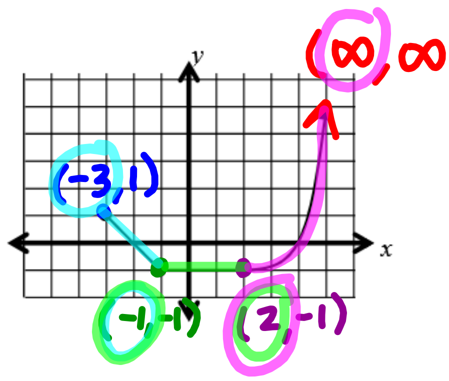
Decreasing: $(-3, -1)$ $-3 < x < -1$

Constant: $(-1, 2)$ $-1 < x < 2$

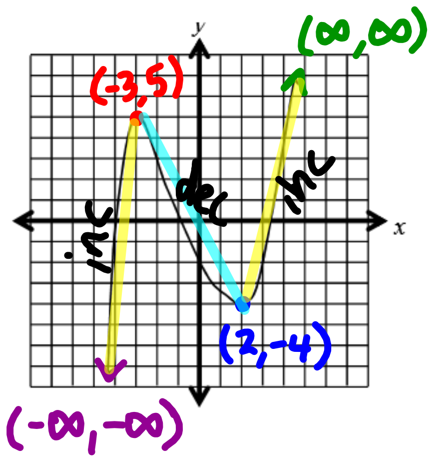
End Behavior:

As $x \to \infty$, $y \to$ ∞.

As $x \to -\infty$, $y \to$ n/a.



Example 3: On the right is the graph of $h(x)$. Describe the following:



Increasing: $(-\infty, -3)$ $(2, \infty)$

Decreasing: $(-3, 2)$

Constant: never

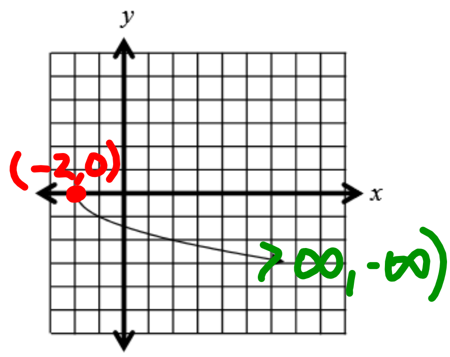
End Behavior:

As $x \to \infty$, $y \to$ ∞.

As $x \to -\infty$, $y \to$ $-\infty$.

Note: If an end stops (like a square root) the x's for that end can't approach infinity, and we just put N/A for what y approaches.

Example 4: On the right is the graph of $m(x) = -\sqrt{x+2}$.
Describe the following:



Increasing: never

Decreasing: $(-2, \infty)$

Constant: never

End Behavior:

As $x \to \infty$, $y \to$ $-\infty$.

As $x \to -\infty$, $y \to$ n/a.

Key Point:
Increasing/decreasing goes from left to right. End behavior does NOT!