11-4 Study Guide and Intervention

Geometric Series

Geometric Series A geometric series is the indicated sum of consecutive terms of a geometric sequence.

The sum S_n of the first n terms of a geometric series is given by

 $S_n = \frac{a_1(1-r^n)}{1-r}$ or $S_n = \frac{a_1 - a_1r^n}{1-r}$, where $r \neq 1$.

Example 1 Find the sum of the first four terms of the geometric sequence for which $a_1 = 120$ and $r = \frac{1}{3}$.

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_4 = \frac{120\left(1 - \left(\frac{1}{3}\right)^4\right)}{1 - \frac{1}{3}} \qquad n = 4, \ a_1 = 120, \ r = \frac{1}{3}$$

$$\approx 177.78 \qquad \text{Use a calculator.}$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \qquad \text{Sum formula}$$

$$S_7 = \frac{\frac{4}{3}(1 - 3^7)}{1 - 3} \qquad n = 7, \ a_1 = \frac{4}{3}, \ r = 3$$

$$\approx 1457.33 \qquad \text{Use a calculator.}$$

The sum of the series is 177.78.

Example 2 Find the sum of the geometric series $\sum_{j=1}^{7} 4 \cdot 3^{j-2}$.

Since the sum is a geometric series, you can use the sum formula.

$$S_n = \frac{a_1(1-r^n)}{1-r}$$
 Sum

$$S_7 = \frac{\frac{4}{3}(1-3^7)}{1-3}$$
 $n = 7, a_1 = \frac{4}{3}, r = \frac{4}{3}$

The sum of the series is 1457.33.

Exercises

Find S_n for each geometric series described.

$$(1.)a_1 = 2, a_n = 486, r = 3$$

1.
$$a_1 = 2, a_n = 486, r = 3$$
 2. $a_1 = 1200, a_n = 75, r = \frac{1}{2}$ 3. $a_1 = \frac{1}{25}, a_n = 125, r = 5$

4.
$$a_1 = 3, r = \frac{1}{3}, n = 4$$

4.
$$a_1 = 3, r = \frac{1}{3}, n = 4$$
 5. $a_1 = 2, r = 6, n = 4$ **6.** $a_1 = 2, r = 4, n = 6$

6.
$$a_1 = 2, r = 4, n = 6$$

$$a_1 = 100, r = -\frac{1}{2}, n = 5$$

8.
$$a_3 = 20$$
, $a_6 = 160$, $n = 8$

7.
$$a_1 = 100, r = -\frac{1}{2}, n = 5$$
 8. $a_3 = 20, a_6 = 160, n = 8$ 9. $a_4 = 16, a_7 = 1024, n = 10$

Find the sum of each geometric series.

10.
$$6 + 18 + 54 + \dots$$
 to 6 terms

$$(11)\frac{1}{4} + \frac{1}{2} + 1 + \dots$$
 to 10 terms

12.
$$\sum_{j=4}^{8} 2^{j}$$

$$\sum_{k=1}^{7} 3 \cdot 2^{k-1}$$

Practice

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Geometric Series

Find S_n for each geometric series described.

1.
$$a_1 = 2$$
, $a_6 = 64$, $r = 2$

3.
$$a_1 = -3$$
, $a_n = -192$, $r = -2$

5.
$$a_1 = -3$$
, $a_n = 3072$, $r = -4$

7.
$$a_1 = 5, r = 3, n = 9$$

9.
$$a_1 = -6, r = -3, n = 7$$

11.
$$a_1 = \frac{1}{3}, r = 3, n = 10$$

2.
$$a_1 = 160, a_6 = 5, r = \frac{1}{2}$$

4.
$$a_1 = -81$$
, $a_n = -16$, $r = -\frac{2}{3}$

6.
$$a_1 = 54$$
, $a_6 = \frac{2}{9}$, $r = \frac{1}{3}$

8.
$$a_1 = -6, r = -1, n = 21$$

10.
$$a_1 = -9, r = \frac{2}{3}, n = 4$$

12.
$$a_1 = 16, r = -1.5, n = 6$$

Find the sum of each geometric series.

13.
$$162 + 54 + 18 + \dots$$
 to 6 terms

14.
$$2 + 4 + 8 + \dots$$
 to 8 terms

15.
$$64 - 96 + 144 - \dots$$
 to 7 terms

16.
$$\frac{1}{9} - \frac{1}{3} + 1 - \dots$$
 to 6 terms

17.
$$\sum_{n=1}^{8} (-3)^{n-1}$$

18.
$$\sum_{n=1}^{9} 5(-2)^{n-1}$$

18.
$$\sum_{n=1}^{9} 5(-2)^{n-1}$$
 19. $\sum_{n=1}^{5} -1(4)^{n-1}$

20.
$$\sum_{n=1}^{6} \left(\frac{1}{2}\right)^{n-1}$$

21.
$$\sum_{n=1}^{10} 2560 \left(\frac{1}{2}\right)^{n-1}$$

22.
$$\sum_{n=1}^{4} 9\left(\frac{2}{3}\right)^{n-1}$$

Find the indicated term for each geometric series described.

23.
$$S_n = 1023$$
, $a_n = 768$, $r = 4$; a_1

24.
$$S_n = 10,160, a_n = 5120, r = 2; a_1$$

25.
$$S_n = -1365, n = 12, r = -2; a_1$$

26.
$$S_n = 665, n = 6, r = 1.5; a_1$$

- 27. CONSTRUCTION A pile driver drives a post 27 inches into the ground on its first hit. Each additional hit drives the post $\frac{2}{3}$ the distance of the prior hit. Find the total distance the post has been driven after 5 hits.
- 28. COMMUNICATIONS Hugh Moore e-mails a joke to 5 friends on Sunday morning. Each of these friends e-mails the joke to 5 of her or his friends on Monday morning, and so on. Assuming no duplication, how many people will have heard the joke by the end of Saturday, not including Hugh?